Assimilation with Different Working Skill Acquisition *

Zijian Qi†

July 10, 2020

Abstract

Discrimination have different forms across places with variety of population composition. We construct a two-stage assimilation model to analyze the discrimination level in groups with different discount factors. I have three main results: First, there always exist an equilibrium for any discount factors and minority group size, the equilibrium will have a on-path action profile with a cutoff rule; second, as group size increase, both discrimination level and the ability cutoff will increase; third, when discount factors varies across different regimes, the effect is not monotonic.

Keywords: Discrimination, Assimilation, Discount factor, Minority, Network

*I would like to thank my committee members Professor Jon X. Eguia, Professor Arijit Mukherjee, and Professor Hanzhe Zhang, along with Christian Cox and my cohorts, for their insightful comments and generous help. All errors are my own.

†Department of Economics, Michigan State University; qizijian@msu.edu

1 Introduction

Discrimination between different groups are commonly seen around the world. During the first half of 20th century, a lot of European immigrants went to US and they experienced little discrimination. On the contrary, before WWII, Germany government discriminate Jews to the extreme and the government wanted to eliminate them from earth. Why the discrimination level differed so much in different countries is a very interesting topic.

Recent literature are most focused on the discrimination against people with low average working skill levels, but seldom study the discrimination against people with high average working skills. For an example, the acting White phenomenon is well studied by Eguia (2017) and Advani and Reich (2015). Both papers proposed a 2-stage game model: for the first paper, the agent of advantaged group will choose a discrimination level in the first stage, and in the second stage, all agents choose a skill level and agents from disadvantaged group will choose their self identity; for the second paper, in the first stage, agents form the minority group will choose their identity and in the second stage, all agents will choose their skill level. In both models, individuals from minority group face a trade-off between cultural and economic incentives: assimilation will gain economic benefit and non-assimilation will prevent the cost. These studies explained the discrimination against minority with lower average working skill level.

However, discrimination against minority with high working skill level do exist. For an example, Asian American has higher working skill level than White people. The positive and negative dichotomy of Asian American stereotypes has been well documented (Fiske, Cuddy, Glick, and Xu 2002, Gilbert 1951, Ho and Jackson 2001, Jackson, Hodge, Gerard, Ingram, Ervin, and Sheppard 1996, Karlins, Coffman, and Walters 1969, Katz and Braly 1933). They are stereotyped as intelligent, industrious, hard working, self-disciplined, good at math and sciences (implying competence), but quiet, shy, unpopular, reserved, traditional, and placing less value on a leisurely life. With that said, Lai and Babcock (2013) studied how White male and female evaluators perceive an Asian American versus White job candidate on the dimensions of competence and social skills and how these perceptions affect evaluators' decisions in hiring and promotion. They found that female evaluators were less likely to select Asian than White candidates into positions involving social skills and were less likely to promote Asian than White candidates into these types of positions. These studies give us an example of discrimination against minority with high working skill level.

To understand the discrimination between different groups, the "self identity" is a very important concept. Akerlof and Kranton (2000) pointed out the important relationship between self identity and economic outcomes. The choice of self identity affects the utility function of the agent, so the choice changes the payoff of the agent herself as well as the payoff of other agents. Furthermore, the collective choice of self identity may change the social norms, affecting identity-based preferences. It is important because the discrimination against people is not a discrimination against race, but also a discrimination against group choice. A person who was born in a family of minority group can still choose the majority as "self identity", thus share the same culture with the majority group.

The empirical results proves that "self identity" play an important role in the utility function. Benjamin, Choi, and Strickland (2010) conducted experiments to show that the social identity of an agent can affect her preference. The discount factor and propensity to save are affected by the choice of majority group. Making Asian American subjects ethnicity salient causes them to exhibit more patient preferences. Making race salient to black subjects decreases discount rates among those who have longstanding roots in the United States. There is also suggestive evidence that native blacks become more risk averse and whites more patient when their racial identity is salient.

To understand the discrimination among groups, we need to figure out why there are working skill level differences between different groups and how do "self identity" affect our utility. I model the difference of working skill level as a result of difference of discount factors among groups. The "self identity" will affect the utility function because there is network effect within groups.

Discount factor is a generalized factor. There are many estimations about the discount factor and the results are very different. The estimation conducted by Hausman (1979), Moore and Viscusi (1990) , Dreyfus and Viscusi (1995), Pender (1996), Coller and Williams (1999), Harrison, Lau, and Williams (2002) ranges from 0.53 to 0.99. By comparing Pender (1996) and Harrison, Lau, and Williams (2002), the first estimates the discount factor in India and the second estimates the discount factor in Denmark. The first one get a result between 0.59 to 0.79 and the second on get a result of 0.78. It is clear that different groups have different discount factors.

The discount factor in our model can be explained in many ways. The discount factor can be think of as the weight putting into the future. It can also be explained as a longer expected life since the longer the life is, the more weight an agent will put in the future utility. It is also a factor of cultural norm. It is shown before that Asia American placing less value in leisure, so the discount factor will be larger for Asia American.

There is evidence that network play an important role in choice of assimilation. Verdier and Zenou (2017) studied the relationship between social network and cultural assimilation. They showed that agents in the center of network have more incentive to assimilate than the agents in the marginal of the network. They also showed that with a denser network (interaction between agents are strong), more people choose to assimilate.

The utility of an agent depend on the average working skill of her group and also depend on how large the group is. The larger the group is, the more benefit that an agent can derive from being a member. Currarini, Jackson, and Pin (2009) found three important results: first, larger groups (measured as a fraction of the population of their respective schools) form a greater fraction of their friendships with people of their same type; second, larger groups form significantly more friendships per capita, that is, members of a group that comprise a small minority in a school form roughly six friendships per capita, while members of groups that comprise large majorities (close to 100 percent of a school) form on average more than eight friendships; third, groups tend to form same-type friendships at rates that exceed the relative fractions in the population. This give us a solid foundation that the utility function will depend on how large the group is.

Our analysis proceeds as follows. Section 2 will setup the model with network effect and two groups have different discount factors. Section 3 solves the game. Section 4 solves the game for an explicit functional form. Section 5 will do the comparative status which explain the main result. Section 6 proposed some testable results. Section 7 discussed about some further extensions. Section 8 concludes the paper.

2 Model Setup

2.1 Players

Consider a society with a continuum of agents. Each agent is identified by her background and her ability. Assume that the set of possible backgrounds is $\{\mathcal{A}, \mathcal{I}\}$, where $\mathcal{A}$ represents the majority group, and $\mathcal{I}$ the minority group. Assume that the set of possible abilities is $[0, 1]$. Let $N = \{\mathcal{A}, \mathcal{I}\} \times [0, 1]$ denote the set of players. For each background $\mathcal{J} \in \{\mathcal{A}, \mathcal{I}\}$, let $N_{\mathcal{J}} = \mathcal{J} \times [0, 1]$denote the subset of agents with background $\mathcal{J}$.

Assume the measure of N, denoted $m(N)$, is equal to 1, and that the distribution of agents is uniform over N. Also assume that $m(N_{\mathcal{I}}) = m$ ($m \in \left(0, \frac{1}{2}\right)$) and $m(N_{\mathcal{A}}) = 1 - m$. That is, the majority group has more population than the minority group. The distribution of ability conditional on background $\mathcal{J}$ is uniform over $[0, 1]$.

For any $i \in N$, let $\theta_i \in [0, 1]$ denote the ability of agent i. Individual ability is private information.

2.2 Lifetime of the Agent

The agent will have two stages. In stage 1, the agent will be considered as young, and she will spend time learning working skills s_i and enjoy her leisure time. In stage 2, the agent will become an adult and will make a choice of assimilation. The agent will then working and payoff accrue.

2.2.1 Skill Level

Agents acquire working skills when they are young. We normalize the time agents have to 1 when they are young. For any agent $i \in N$, she can choose her leisure time l_i and spend the rest of time $1 - l_i$ learning. Based on the ability θ_i, the working skill level agent i can get is $s_i = \theta_i(1 - l_i)$.

2.2.2 Discount Factor

Agents with different backgrounds have different time preferences. Assume that agents with background $\mathcal{A}$ have discount factor $\beta_{\mathcal{A}}$ and agents with background $\mathcal{I}$ have discount factor $\beta_{\mathcal{I}}$. I assume that $\beta_{\mathcal{A}} < \beta_{\mathcal{I}} < 1$ in this paper, that is, the minority group put more weight on the future utility.

2.3 Choice of Social Group

Assume that there are two self identity groups A and I, characterized by two sets of social norms and actions expected from their members. In-group networks are strong and the networks across groups are very small.

In stage 2, assume that agents with background $\mathcal{A}$ will identify them self as A, $N_{\mathcal{A}} \subseteq A$. Assume that any agent with background $\mathcal{I}$ can choose to belong to social group I at no cost, or she can embrace the cultural norms of group A to then join A. Let $a_i \in \{0, 1\}$ be the choice of agent $i \in N_{\mathcal{I}}$. Let $a_i = 0$ denote that $i \in N_{\mathcal{I}}$ chooses to be part of group I and not to assimilate, and let $a_i = 1$ denote that agent $i \in N_{\mathcal{I}}$ chooses to adopt the majority cultural norms and to become a member of the majority group A. If $a_i = 1$, we say that $i \in N_{\mathcal{I}}$ "assimilates."

2.4 The Cost of Assimilation

The cost of assimilation is d for agent i, where $d \in \mathbb{R}_+$ is the difficulty of assimilation to become a member of A. This difficulty of assimilation d is an endogenous, strategic variable. It can be interpreted as the level of discrimination: if agents with background $\mathcal{A}$ are welcoming to those who assimilate, d is small; if agents with background $\mathcal{A}$ are hostile or if they give a cold shoulder to those who are trying to assimilate, then d is high.

The level of d is chosen endogenously in the model by an agent with background $\mathcal{A}$. In the setup, we assume that agents with background $\mathcal{A}$ collectively choose an agent $h \in N_{\mathcal{A}}$ as a representative, then the agent h will choose the discrimination level d. As we shown in Section 3, all agents with background $\mathcal{A}$ will share the same optimal choice, so the mechanism of chossing the representative h will not affect the equilibrium.

2.5 Network Effect

Agents will benefit from social group network effect. Agents in the same social group share the same behavior and cultural so they will be closely connected. With larger group size, each member in the group will benefit more.

Mathematically, we use a a function $f(m_J)$ to model the network effect. We will assume that $f(0) = 0, f'(.) \geq 0, f'(0) \leq 1, f''(.) \leq 0$. That is, the larger the group is, the greater the network effect. Also, the marginal benefit of network effect is decreasing.

2.6 Timing of the Game

The timing of the game is as follows:

1. For any agent $i \in N$, i chooses the leisure time l_i when they are young and acquires working skill s_i accordingly. All agents will act simultaneously.

2.1 All agents become adults and observe the working skill s_i of other agents. Agents with background $\mathcal{A}$ choose a representative $h \in \mathcal{A}$ and h chooses the discrimination level d.

2.2 All agents observe d. Agents with background $\mathcal{I}$ make an assimilation choice a_i based on the information she have. All agents with background $\mathcal{I}$ will act simultaneously. Payoffs accrue.

2.7 Utility Functions

In stage 1, agent $i \in N$ will derive a utility level $log(l_i)$ for enjoying the leisure time $U_i^1(l_i) = Log(l_i)$.

In stage 2, agents become adults and start working. For each social group $J \in \{A, I\}$, let s_J be the average working skill of agents in J and m_J be the size of the group. Assume that an agent i with skill s_i in social group $J \in \{A, I\}$ with average skill s_J and size m_J derives a utility $f(m_J)s_J s_i$. In addition, agent i may experience costs of assimilation.

Let $U_i^2(d, a_i)$ denote the utility function of agent i in stage 2 as a function of the discrimination level d and the assimilation decisions a_i. We can fix $a_i = 0$ exogenously for any $i \in N_{\mathcal{A}}$, then the utility in stage 2 of an agent i in social group $J \in \{A, I\}$ can be written as:

$$U_i^2(d, a_i) = Log[f(m_J)s_J s_i - a_i d]$$

The agents are impatient and agents with different backgrounds have different discount factors. Let $\beta_{\mathcal{A}}$ denotes the discount factor for agents with background $\mathcal{A}$ and $\beta_{\mathcal{I}}$ denotes the discount factor for agents with background $\mathcal{I}$. I assume that $\beta_{\mathcal{A}} < \beta_{\mathcal{I}} < 1$.

Above all, the utility of an agent i with background $\mathcal{J} \in \{\mathcal{A}, \mathcal{I}\}$ in social group $J \in \{A, I\}$ can be written as:

$$U_i(l_i, d, a_i) = Log(l_i) + \beta_{\mathcal{J}} Log[f(m_J)s_J s_i - a_i d]$$

This completes the definition of game $\Gamma_{m,\beta_{\mathcal{A}},\beta_{\mathcal{I}}} = (N, S, U)$.

3 Solution to the Game

I will solve the game by backward induction. In stage 1, every agent i need to choose her leisure time l_i. In stage 2, the representative agent h with background $\mathcal{A}$ will choose the discrimination level d and every agent with background $\mathcal{I}$ will choose the assimilation action a_i.

Using backward induction, I will first characterize how agents make the assimilation decision in stage 2. We will then find the best choice of discrimination level d. After solving these, we will characterize the choice of leisure time for all agents.

3.1 Choice of Assimilation

For agents with background I, they make the assimilation choice at the same time. With the proposition below, we can identify the structure of equilibria.

Proposition 1 *For any bounded measurable function s over N, for any discrimination level $d \in \mathbb{R}_+$, there exists $c \in (0, 1]$ and $p \in [0, 1]$ such that*

$$a_i(d, s_i) = \begin{cases} 1 & \textit{if } s_i > c \textit{ or } s_i = c \textit{ with probability } p; \\ 0 & \textit{if } s_i < c \textit{ or } s_i = c \textit{ with probability } (1-p); \end{cases}$$

constitutes an equilibrium.

The proposition guarantees the existence of equilibrium but not the uniqueness. In general, the uniqueness in stage 2 cannot be guaranteed since the distribution of working skills s over all agents N can be any function.

I focus on one specific functional form of the distribution of working skills s, which would be our on-path equilibrium result. The working skill distribution will take the form of

$$s_i = \begin{cases} \alpha_{\mathcal{A}}\theta_i & \text{if } i \in N_{\mathcal{A}}; \\ \alpha_{\mathcal{I}}\theta_i + s^0 & \text{if } \theta_i \geq \theta^0 \text{ and } i \in N_{\mathcal{I}}; \\ \alpha_{\mathcal{I}}\theta_i & \text{if } \theta_i < \theta^0 \text{ and } i \in N_{\mathcal{I}} \end{cases}$$

for some $\alpha_{\mathcal{A}}, \alpha_{\mathcal{I}} \in (0,1)$, $s^0 \in [0,1]$, $\theta^0 \in [0,1]$. Denote S be the set that contains all possible working skill distribution in this functional form.

Corollary 1 *For any working distribution $s \in S$, for any discrimination level $d \in \mathbb{R}_+$, there exists $c \in (0,1]$ such that*

$$a_i(d, s_i) = \begin{cases} 1 & \textit{if } s_i \geq c; \\ 0 & \textit{if } s_i < c \end{cases}$$

constitutes an equilibrium.

For this specific functional form, the uniqueness still cannot be guaranteed since it would depend on the functional form of the network effect f. Since there is no point mass in the working skill distribution, the structure of equilibrium can be pinned down to the form above. The cutoff strategy simplifies our analysis because the representative of $N_{\mathcal{A}}$ can indirectly choose the cutoff θ^c by directly choosing the discrimination level d.

Denote $C(s, d)$ be a correspondence, such that for every element $c \in C(s, d)$, the action profile $a_i(d, s_i)$ with cutoff point c constitutes an equilibrium for working skill distribution $s \in S$ and discrimination level $d \in \mathbb{R}_+$.

3.2 Choice of Discrimination Level

The choice of discrimination level d is determined by the representative agent h with background $\mathcal{A}$. The representative agent faces the problem:

$$\max_{d \in [0,\infty)} f(m_A) s_A s_h$$

For agent h, s_h is fixed so the maximization problem would be the same as:

$$\max_{d\in[0,\infty)} f(m_A)s_A$$

For any agent $i \in N_{\mathcal{A}}$, the utility maximization problem will be the same. That is, all agents with background $\mathcal{A}$ share one preference profile. I assume that the representative h is random chosen from all agents with background $\mathcal{A}$. The choice of d will not be affected by the choice of representative h. Thus, the mechanism of choosing h will not affect the equilibrium, as I discussed before.

Given that direct choice of d will indirectly choose θ^c, we can treat the utility maximization problem of h as:

$$\max_{\theta\in[0,1]} f(m_A)s_A$$

Proposition 2 *For any working skill distribution $s \in S$. There is a unique discrimination level d^* along with a cutoff $c^* \in C(s, d^*)$ such that, the representative h choose discrimination level d^* and agents with background $\mathcal{I}$ choose action profile*

$$a_i(d^*, s_i) = \begin{cases} 1 & \text{if } s_i \geq c^*; \\ 0 & \text{if } s_i < c^* \end{cases}$$

constitutes an equilibrium, and θ^* maximizes $f(m_A)s_A$.

3.3 Choice of Working Skill

When choosing working skills, I assume that agents are sequentially rational, update their beliefs according to Bayes rule. We can characterize the equilibrium as follow:

Proposition 3 *At any equilibrium, $s_i = \frac{\beta_{\mathcal{J}}}{1+\beta_{\mathcal{J}}}\theta_i$ for any agent i in group J with $a_i = 0$ and $s_i = \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta_i + s^*$ for any agent with $a_i = 1$, where $s^* = \frac{d^*}{(1+\beta)f(m_A)s_A}$ is a constant.*

By the proposition, at any equilibrium, the on-path choice of working skill will be in the set S. This coincide with my definition of S. We cannot expand S to the set of any function on N since we need measurable functions to calculate the average working skill level.

Proposition 4 *There exist an equilibrium for the game $\Gamma_{m,\beta_{\mathcal{A}},\beta_{\mathcal{I}}}$.*

The uniqueness of equilibrium cannot be guaranteed and it will highly depend on the functional form of f. In the next section, I will solve the game with a specific f.

4 Explicit Function Form

I assume that $f(m) = m - \frac{1}{2}m^2$ as an explicit function of f. We can easily check that this function satisfies the assumptions above. We will solve the equilibrium for this specific case.

Proposition 5 *For $f(m) = m - \frac{1}{2}m^2$ and any $\beta_{\mathcal{A}}, \beta_{\mathcal{I}} \in (0,1)$, $m \in (0, \frac{1}{2})$, there exist (s^*, θ^*) such that*

$$\begin{aligned}
&\left[\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}(1+m\theta^*)(1-m) + \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}m(1+m\theta^* - 3\theta^{*2}) + 2ms^*(1+m\theta^*)(1-\theta^*)\right]\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta^* \\
\leq &\left[\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}(1+m\theta^*)(1-m) + \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}m(1+m\theta^*)(1-\theta^{*2}) + 2ms^*(1+m\theta^*)(1-\theta^*)\right](1+\beta_{\mathcal{I}})s^*
\end{aligned}$$

$$\begin{aligned}
&\left[\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}(1+m\theta^*)(1-m) + \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}m(1+m\theta^* - 3\theta^{*2}) + 2ms^*(1+m\theta^*)(1-\theta^*)\right]\left[\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta^* + s^*\right] \\
\geq &\left[\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}(1+m\theta^*)(1-m) + \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}m(1+m\theta^*)(1-\theta^{*2}) + 2ms^*(1+m\theta^*)(1-\theta^*)\right](1+\beta_{\mathcal{I}})s^*
\end{aligned}$$

and $\theta^* \in [\theta^1, \theta^2]$ where

$$\theta^1 = \frac{-\left[\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}} + 2ms^*\right] + \sqrt{(\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}} + 2ms^*)^2 + 3m\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\left[\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}(1-m) + \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}m - 2s^*(1-m)\right]}}{3m\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}}$$

$$\theta^2 = \frac{-\left[\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}} + ms^*\right] + \sqrt{(\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}} + ms^*)^2 + 3m\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\left[\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}(1-m) + \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}m + 2ms^*\right]}}{3m\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}}$$

. *There exists an equilibrium that related to* (s^*, θ^*) *and the on-path action profile would be*

$$s_i = \begin{cases} \frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}\theta_i & \text{if } i \in N_{\mathcal{A}}; \\ \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta_i + s^* & \text{if } \theta_i \geq \theta^* \text{ and } i \in N_{\mathcal{I}}; \\ \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta_i & \text{if } \theta_i < \theta^* \text{ and } i \in N_{\mathcal{I}} \end{cases}$$

$$d^* = \left[\frac{\beta_{\mathcal{A}}}{4(1+\beta_{\mathcal{A}})}(1+m\theta^*)(1-m) + \frac{\beta_{\mathcal{I}}}{4(1+\beta_{\mathcal{I}})}m(1+m\theta^*)(1-\theta^{*2}) + \frac{1}{2}ms^*(1+m\theta^*)(1-\theta^*)\right](1+\beta_{\mathcal{I}})s^*$$

$$a_i(s_i) = \begin{cases} 1 & \text{if } s_i \geq \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta^* + s^*; \\ 0 & \text{if } s_i < \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta^* \end{cases}$$

The proposition characterizes the equilibrium and provides a way to calculate it.

5 Comparative Status

Based on the equilibrium calculated in section 4, now I will talk about some comparative status of the equilibrium. With explicit functional form of the network effect, the equilibrium is not unique. For every pair of parameters $(\beta_{\mathcal{A}}, \beta_{\mathcal{I}}, m)$, I calculated $(d^{min}, d^{max}, \theta^{min}, \theta^{max})$. Note that in some conditions, there may exist an equilibrium that two groups are totally separated and no assimilation will happen. This equilibrium is not the main focus of this paper so the discussion below will not consider this equilibrium.

I calculated the minimum and maximum of both ability cutoff (θ^*) and the discrimination level (d^*) for some parameters. For every pair of parameters $(\beta_{\mathcal{A}}, \beta_{\mathcal{I}}, m)$, every discrimination level $d \in [d^{min}, d^{max}]$ can be achieved by some equilibrium. Similarly, every ability cutoff

$\theta \in [\theta^{min}, \theta^{max}]$ can be achieved by some equilibrium, but (d^{min}, θ^{max}) or (d^{max}, θ^{min}) may not be achieved by some equilibrium. That is, the area in $\mathbb{R}^2$ that the pair (d, θ) can be achieved is not be a rectangle.

5.1 Group Size

First I will talk about the effect of group size. By comparing Figure 1 to Figure 4, in general, the ability cutoff θ^* is increasing as the group size become larger. When $\beta_{\mathcal{A}}$ is small, the increasing of ability cutoff is very significant on both θ^{min} and θ^{max}, and when it is large, θ^{min} and θ^{max} is still increasing but the slope is much smaller. When $\beta_{\mathcal{A}}$ is small, according to proposition 5, the average working skill level of agents with background $\mathcal{A}$ is low. When the group size of the minority group is small, the majority group would like almost all minority people to assimilate since the minority have higher working skill level, so the the ability cutoff is very small; when the group size of minority group becomes larger, the assimilated minority will increase the average working skill level of group A, so the ability cutoff becomes larger. The increasing of ability cutoff is significant when $\beta_{\mathcal{A}}$ is small since the increase of average working skill of group A is very large due to assimilation. The increase of average working skill of group A is small when $\beta_{\mathcal{A}}$ is large and in that case, both θ^{min} and θ^{max} increased slowly when the group size of minority (m) increase.

Then we focus on the discrimination level. By comparing Figure 13 to Figure 16, in general both d^{min} and d^{max} increases as the group size (m) increase. When $\beta_{\mathcal{A}}$ is small, d^{max} increased significantly and when $\beta_{\mathcal{A}}$ is large, d^{max} increased slowly. This may have a similar reason as explained above. When the ability cutoff is small (in general), the discrimination level will be small when the ability cutoff is large, the discrimination level will be large accordingly. The increasing speed of d^{max} is similar as the increasing speed of θ^*. Another result would be that the increasing of d^{min} is significant only in Figure 14 where the difference between the discount factors is very large.

5.2 Different Discount Factors

Different discount factors will affect the ability cutoff and the discrimination level at the same time. By comparing Figure 5 to Figure 12, we can find several results. When $\beta_{\mathcal{I}}$ is fixed, the increasing of $\beta_{\mathcal{A}}$ will result in an increase on both θ^{min} and θ^{max}. On the other way, when $\beta_{\mathcal{A}}$ is fixed, the increasing of $\beta_{\mathcal{I}}$ will result in a decrease on θ^{max} but an increase on θ^{min}. This is a very interesting result. Intuitively, when the difference between discount factors become larger, the ability cutoff will become smaller. In proposition 5, we can see that the on-path action profile is

$$s_i = \begin{cases} \frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}}\theta_i & \text{if } i \in N_{\mathcal{A}}; \\ \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta_i + s^* & \text{if } \theta_i \geq \theta^* \text{ and } i \in N_{\mathcal{I}}; \\ \frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}\theta_i & \text{if } \theta_i < \theta^* \text{ and } i \in N_{\mathcal{I}} \end{cases}$$

There is a discontinuity at $\theta = \theta^*$. Agents with ability above the cutoff will exert extra effort to acquire an extra working skill s^*. In this way, even the difference between the discount factor is very small, the discontinuity s^* will provide some extra working skill level so that the cutoff could be small. When $\beta_{\mathcal{A}}$ is fixed and $\beta_{\mathcal{I}}$ increased, the effect of s^* will dominate so the θ^{min} is increasing as $\beta_{\mathcal{I}}$ increasing. When $\beta_{\mathcal{I}}$ is fixed and $\beta_{\mathcal{A}}$ is increased, the effect of difference of discount factors will dominate so θ^{min} increased as $\beta_{\mathcal{A}}$ increase.

Now I will focus on the discrimination level. By comparing Figure 17 to Figure 24, I can find similar result as last paragraph. When fixed $\beta_{\mathcal{I}}$, both d^{min} and d^{max} increases as $\beta_{\mathcal{A}}$ increase. This is when the effect of difference between discount factors dominates. When $\beta_{\mathcal{A}}$ is fixed, in general, when $\beta_{\mathcal{I}}$ increases, d^{min} will increase and d^{max} will decrease. The increasing of d^{min} is because the effect of s^* (discontinuity of working skill level) dominates. There is another interesting result, that is when $\beta_{\mathcal{A}} = 0.1$, $m = 3$, d^{max} will first increase then decrease. This may be the effect of the combination of two effects.

6 Testable results

6.1 Group Size Change

As explained in last section, when the group size become large, the discrimination level will be higher than the discrimination level when the group size is small. The migration process may serve as an empirical data of this change. When migration just started, the population of minority group in a community is very small, the discrimination against them should be small. As more and more minority people migrate to the community, the discrimination level should larger than it is before. The discrimination level could be captured by the number of conflicts between majority and minority or the number of marriages between majority and minority groups.

6.2 Discount Factor Change

People have different discount factors may experience different discrimination level. Jewish people is minority group in many countries and they share the same culture. Assume they are the minority group and they share the same $\beta_{\mathcal{I}}$ around the world. By comparing the discrimination level of Jews around the world, we should see high discrimination level in countries with high discount factor (culture that put more weights in future utility).

7 Further Discussion

As discussed before, there always exist an equilibrium that two groups remain separated. It is easily to check that when $f(1-m)\frac{\beta_{\mathcal{A}}}{1+\beta_{\mathcal{A}}} \leq f(m)\frac{\beta_{\mathcal{I}}}{1+\beta_{\mathcal{I}}}$, the separation equilibrium exists. The existence of this separation equilibrium will provide more interesting results for the model.

Another extension would be the study of evolution of the group size. With assimilation, group size will change according to time. Different groups will have different growth rates and the speed of assimilation will depend on the difference between discount factors.

A third extension would be use a more general functional form of the network effect. The equilibrium will exist in general but the change of discrimination level and ability cutoff will very across different functional forms.

8 Conclusion

In this paper, we construct a 2-stage game model to explain the difference of discrimination level across different scenarios. There are several main results. First, there exist equilibrium for any discount factors and minority group size, the equilibrium will have a on-path action profile with a cutoff rule. Second, as group size increase, both discrimination level and the ability cutoff will increase. Third, when discount factors varies across different regimes, there are two effects that drive the discrimination level and ability cutoff to opposite directions. When $\beta_{\mathcal{I}}$ is fixed, the larger the difference between discount factors, the larger the discrimination level and ability cutoff. When $\beta_{\mathcal{A}}$ is fixed, the two effects are mixed and there is no general results for the discrimination level and ability cutoff.

References

Advani, Arun and Bryony Reich (2015), "Melting pot or salad bowl: The formation of heterogeneous communities." Technical report, IFS Working Papers.

Akerlof, George A. and Rachel E. Kranton (2000), "Economics and identity." *The Quarterly Journal of Economics*, 115, 715–753.

Benjamin, Daniel J., James J. Choi, and A. J. Strickland (2010), "Social identity and preferences." *The American Economic Review*, 100, 1913–1928.

Coller, Maribeth and Melonie B. Williams (1999), "Eliciting individual discount rates." *Experimental Economics*, 2, 107–127.

Currarini, Sergio, Matthew O. Jackson, and Paolo Pin (2009), "An economic model of friendship: Homophily, minorities, and segregation." *Econometrica*, 77, 1003–1045.

Dreyfus, Mark K. and W. K. Viscusi (1995), "Rates of time preference and consumer valuations of automobile safety and fuel efficiency." *The Journal of Law and Economics*, 38, 79–105.

Eguia, Jon X. (2017), "Discrimination and assimilation at school." *Journal of Public Economics*, 156, 48 – 58.

Fiske, Susan T., Amy J. C. Cuddy, Peter Glick, and Jun Xu (2002), "A model of (often mixed) stereotype content: Competence and warmth respectively follow from perceived status and competition." *Journal of Personality and Social Psychology*, 82, 878–902.

Gilbert, G. M. (1951), "Stereotype persistence and change among college students." *The Journal of Abnormal and Social Psychology*, 46, 245–254.

Harrison, G.W, M.I Lau, and M.B Williams (2002), "Estimating individual discount rates in denmark: A field experiment." *The American Economic Review*, 92, 1606–1617.

Hausman, Jerry A. (1979), "Individual discount rates and the purchase and utilization of energy-using durables." *The Bell Journal of Economics*, 10, 33–54.

Ho, Colin and Jay W. Jackson (2001), "Attitudes toward asian americans: theory and measurement." *Journal of Applied Social Psychology*, 31, 1553.

Jackson, Linda A., Carole N. Hodge, Donna A. Gerard, Julie M. Ingram, Kelly S. Ervin, and Lori A. Sheppard (1996), "Cognition, affect, and behavior in the prediction of group attitudes." *Personality and Social Psychology Bulletin*, 22, 306–316.

Karlins, Marvin, Thomas L. Coffman, and Gary Walters (1969), "On the fading of social stereotypes: Studies in three generations of college students." *Journal of Personality and Social Psychology*, 13, 1–16.

Katz, D. and K. Braly (1933), "Racial stereotypes of one hundred college students." *The Journal of Abnormal and Social Psychology*, 28, 280–290.

Lai, Lei and Linda C. Babcock (2013), "Asian americans and workplace discrimination: The interplay between sex of evaluators and the perception of social skills: Asian americans and workplace discrimination." *Journal of Organizational Behavior*, 34, 310–326.

Moore, Michael J. and W. K. Viscusi (1990), "Models for estimating discount rates for long-term health risks using labor market data." *Journal of Risk and Uncertainty*, 3, 381–401.

Pender, John L. (1996), "Discount rates and credit markets: Theory and evidence from rural india." *Journal of Development Economics*, 50, 257–296.

Verdier, Thierry and Yves Zenou (2017), "The role of social networks in cultural assimilation." *Journal of Urban Economics*, 97, 15–39.

Figures

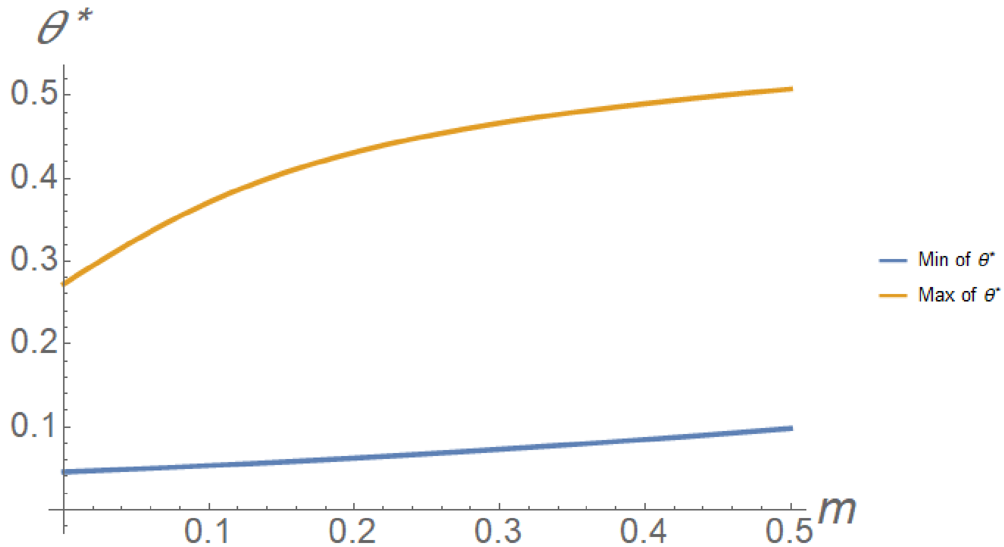

Figure 1: Ability Cutoff ($\beta_{\mathcal{A}} = 0.1, \beta_{\mathcal{I}} = 0.2$)

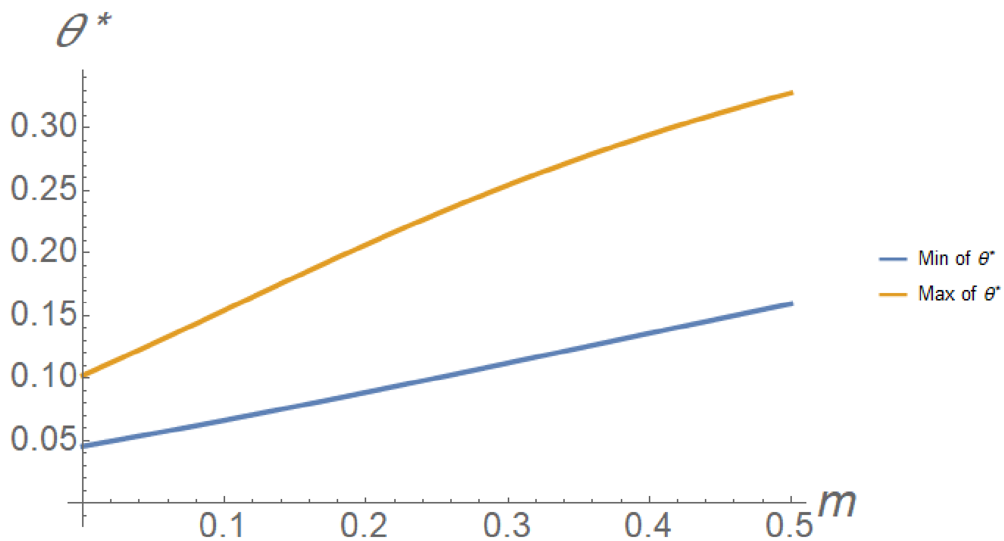

Figure 2: Ability Cutoff ($\beta_{\mathcal{A}} = 0.1, \beta_{\mathcal{I}} = 0.8$)

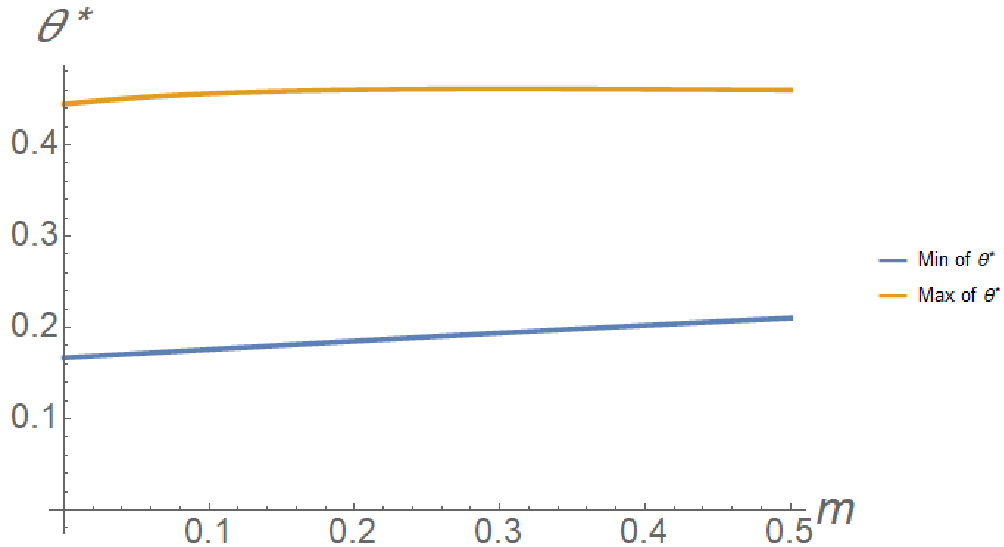

Figure 3: Ability Cutoff ($\beta_{\mathcal{A}} = 0.5, \beta_{\mathcal{I}} = 0.6$)

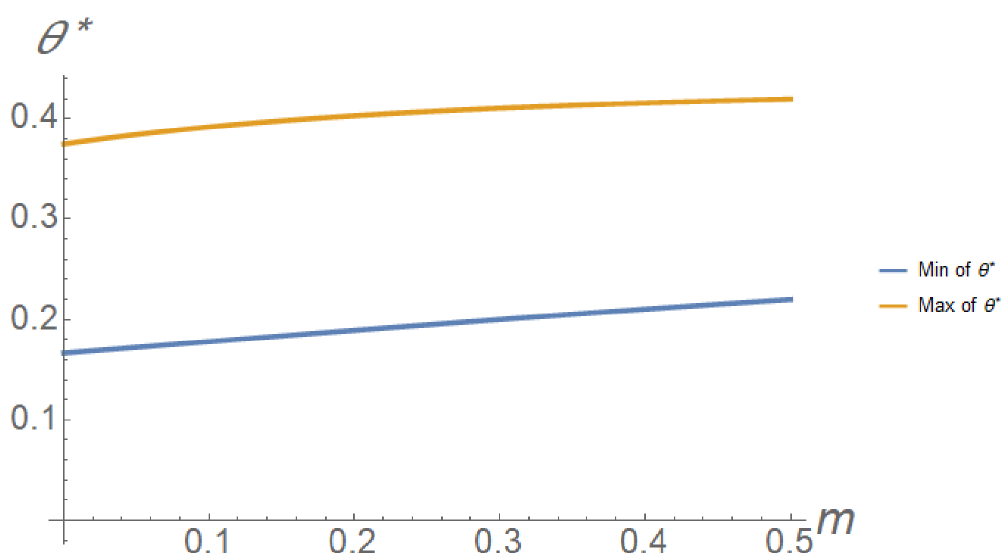

Figure 4: Ability Cutoff ($\beta_{\mathcal{A}} = 0.5, \beta_{\mathcal{I}} = 0.8$)

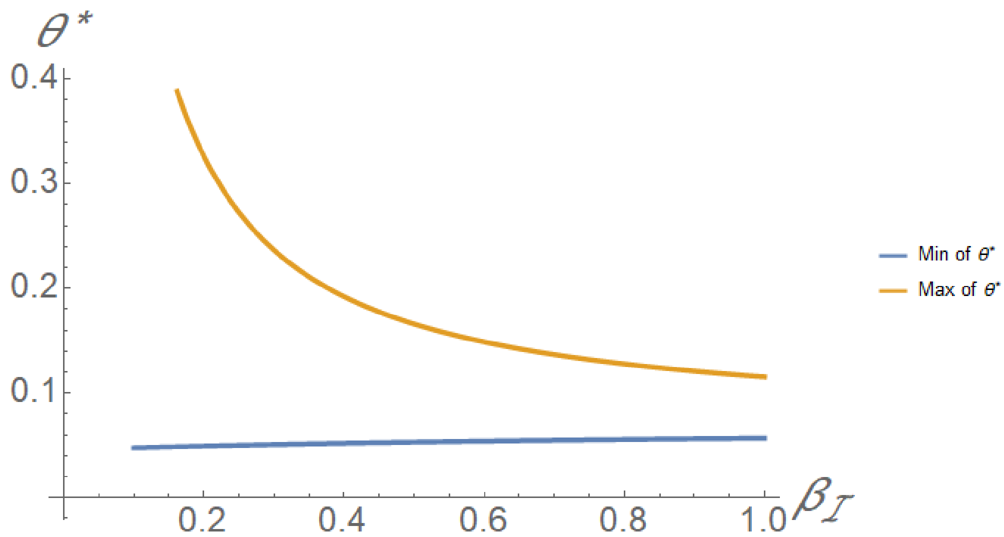

Figure 5: Ability Cutoff ($\beta_{\mathcal{A}} = 0.1, m = 0.05$)

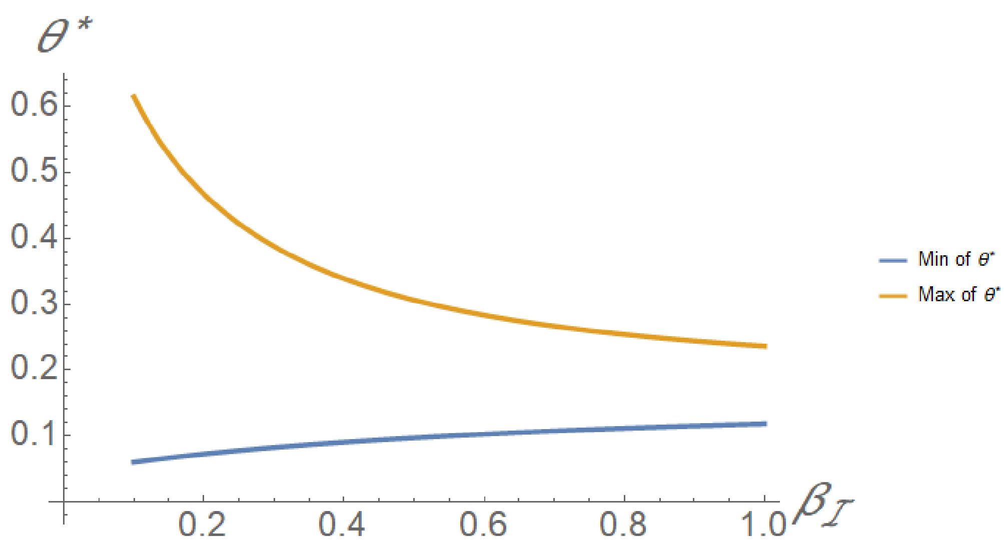

Figure 6: Ability Cutoff ($\beta_{\mathcal{A}} = 0.1, m = 0.3$)

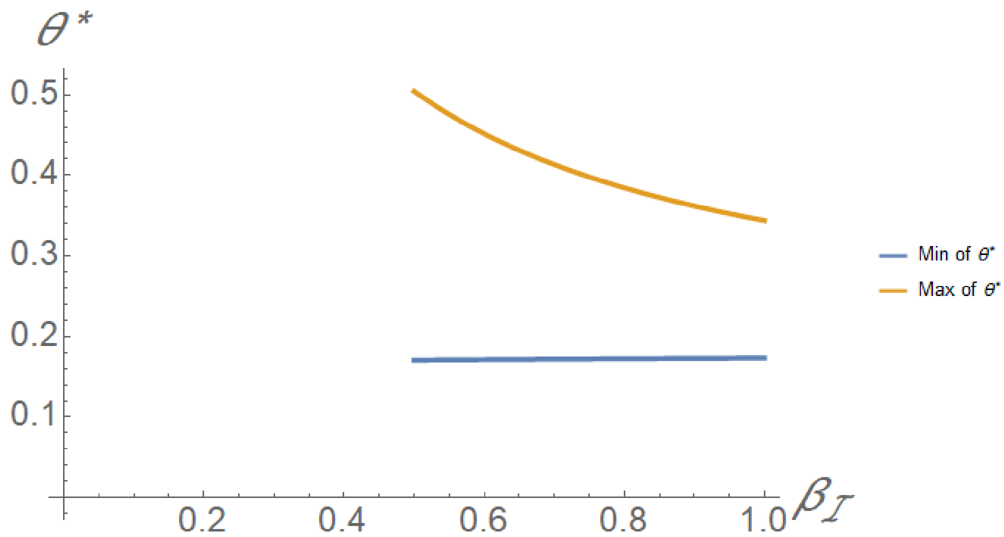

Figure 7: Ability Cutoff ($\beta_{\mathcal{A}} = 0.5, m = 0.05$)

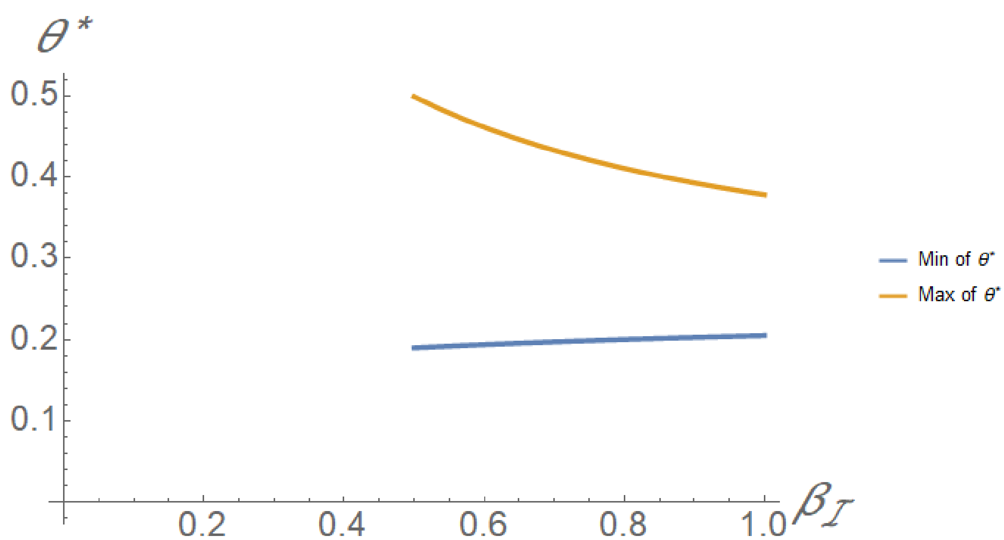

Figure 8: Ability Cutoff ($\beta_{\mathcal{A}} = 0.5, m = 0.3$)

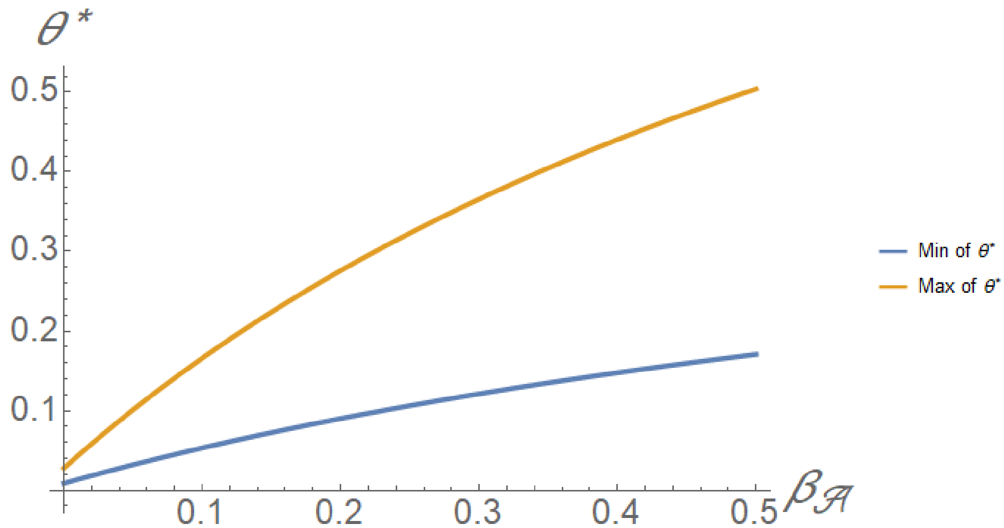

Figure 9: Ability Cutoff ($\beta_{\mathcal{I}} = 0.5, m = 0.05$)

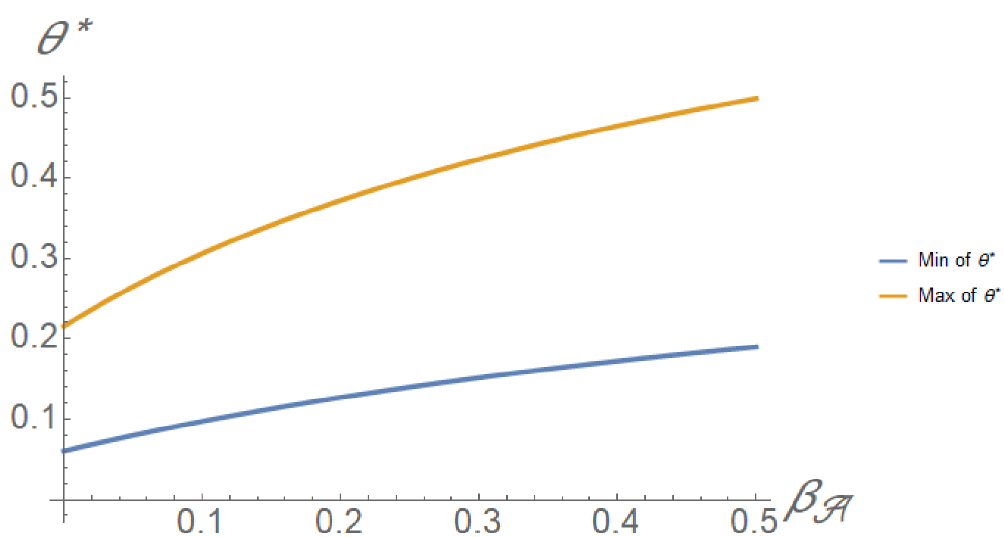

Figure 10: Ability Cutoff ($\beta_{\mathcal{I}} = 0.5, m = 0.3$)

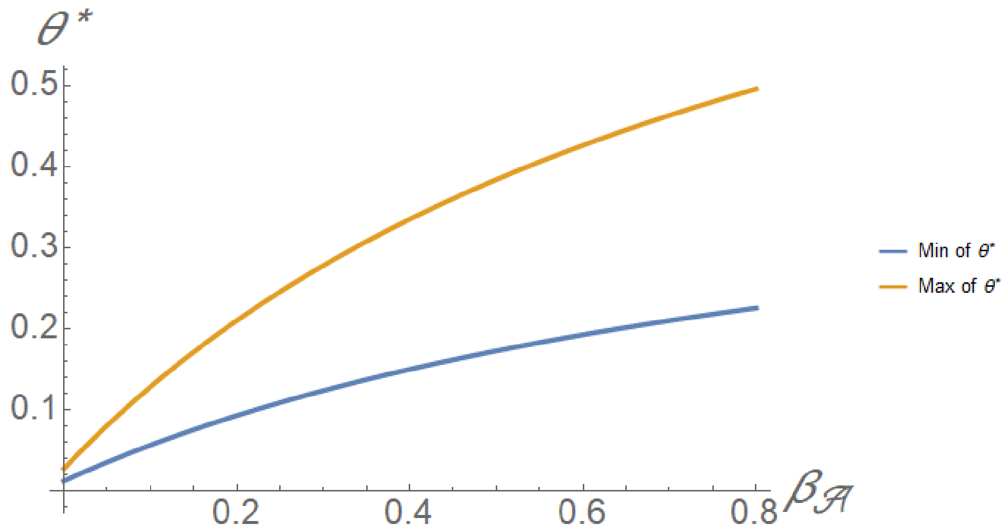

Figure 11: Ability Cutoff ($\beta_{\mathcal{I}} = 0.8, c = 0.05$)

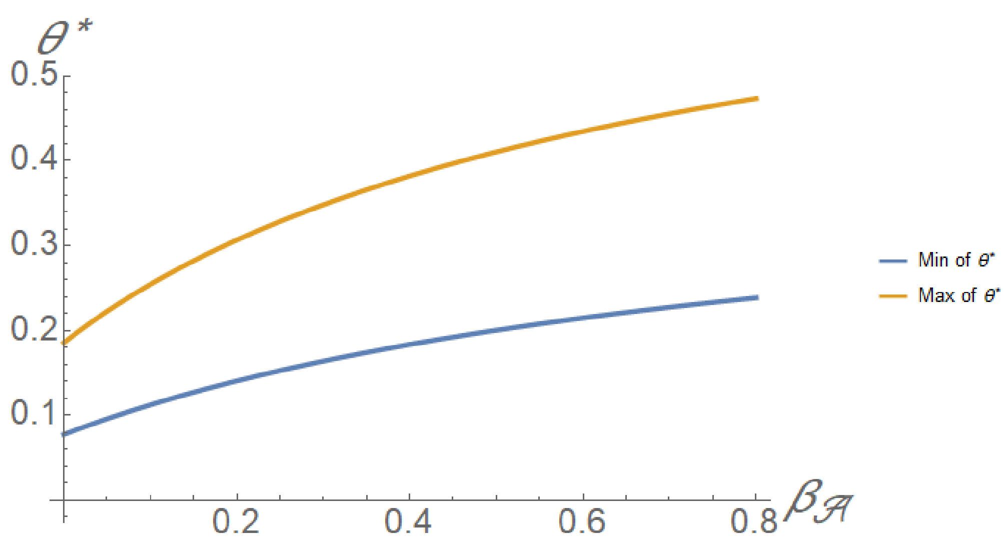

Figure 12: Ability Cutoff ($\beta_{\mathcal{I}} = 0.8, m = 0.3$)

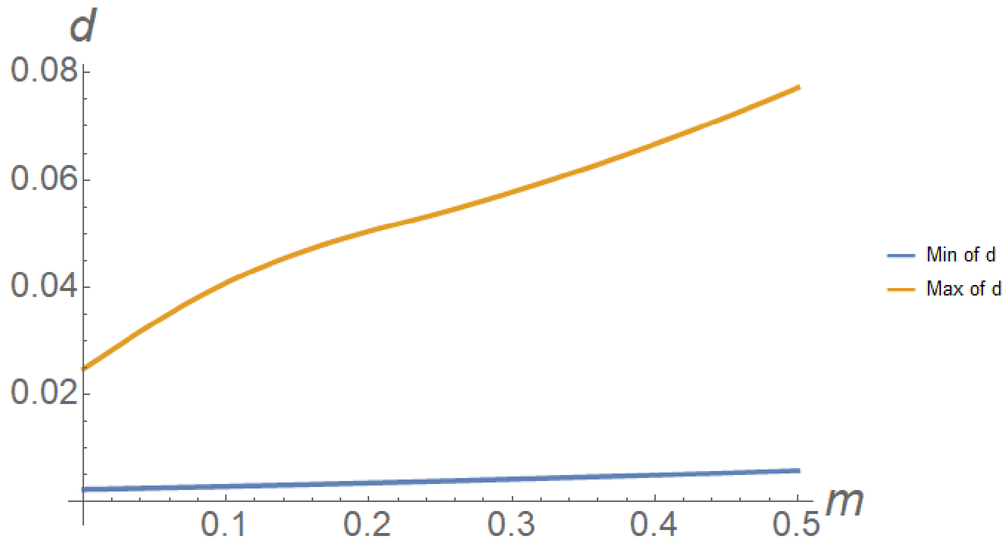

Figure 13: Discrimination Level ($\beta_{\mathcal{A}} = 0.1, \beta_{\mathcal{I}} = 0.2$)

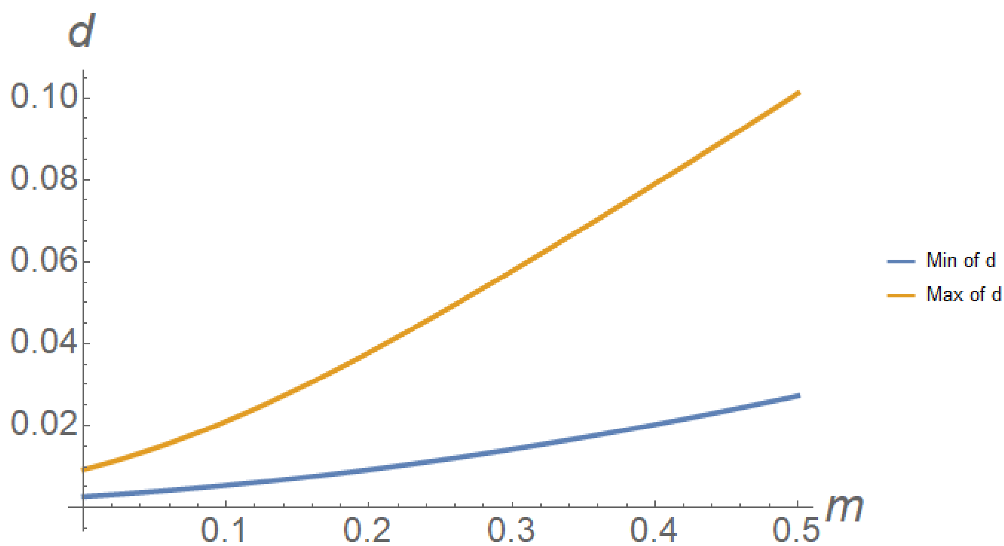

Figure 14: Discrimination Level ($\beta_{\mathcal{A}} = 0.1, \beta_{\mathcal{I}} = 0.8$)

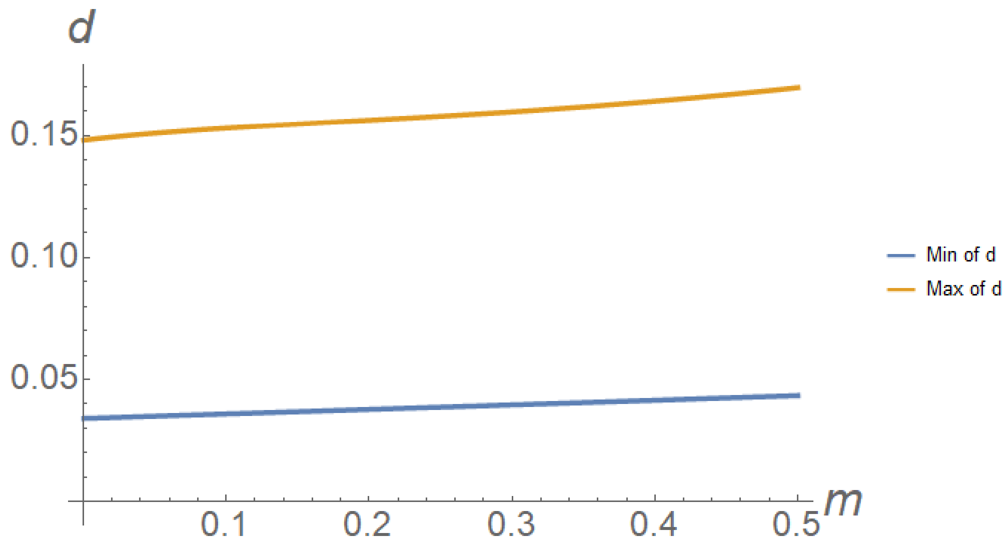

Figure 15: Discrimination Level ($\beta_{\mathcal{A}} = 0.5, \beta_{\mathcal{I}} = 0.6$)

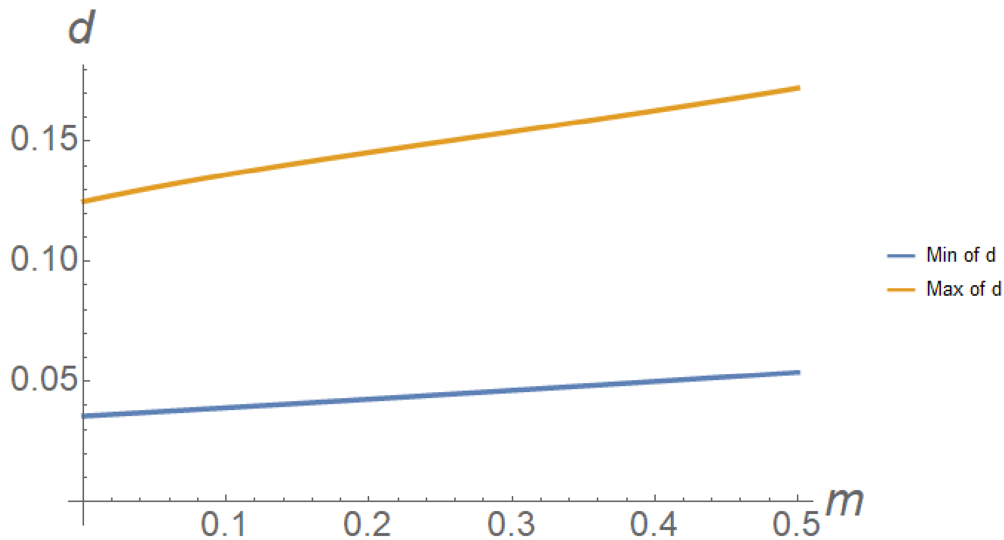

Figure 16: Discrimination Level ($\beta_{\mathcal{A}} = 0.5, \beta_{\mathcal{I}} = 0.8$)

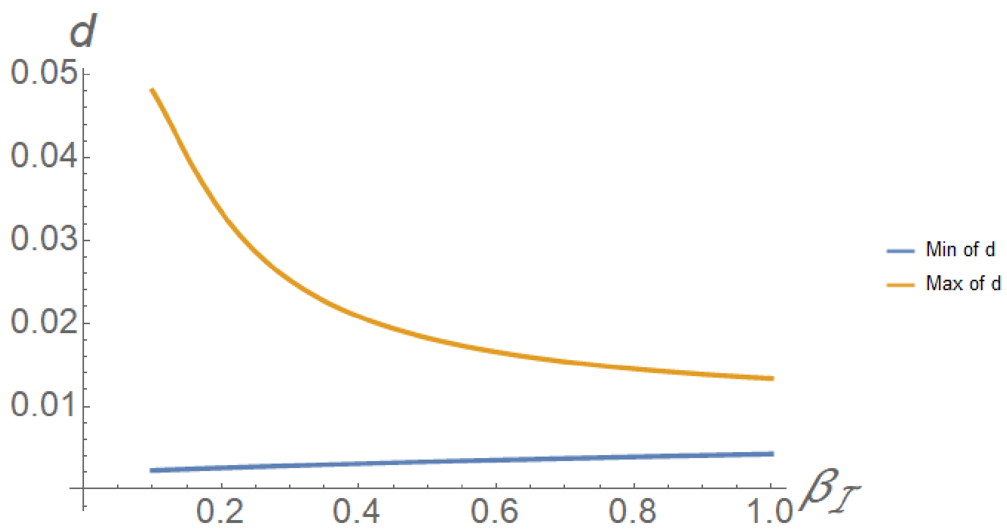

Figure 17: Discrimination Level ($\beta_{\mathcal{A}} = 0.1, m = 0.05$)

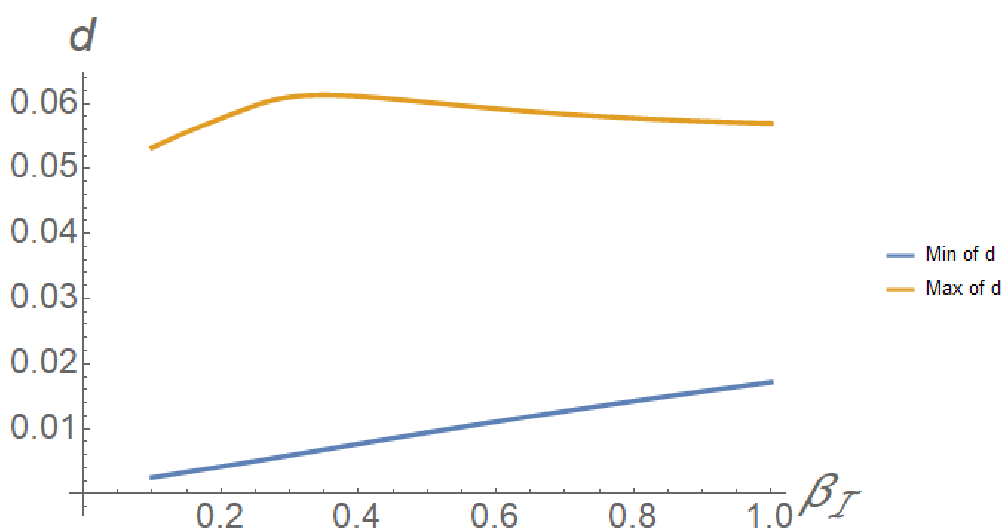

Figure 18: Discrimination Level ($\beta_{\mathcal{A}} = 0.1, m = 0.3$)

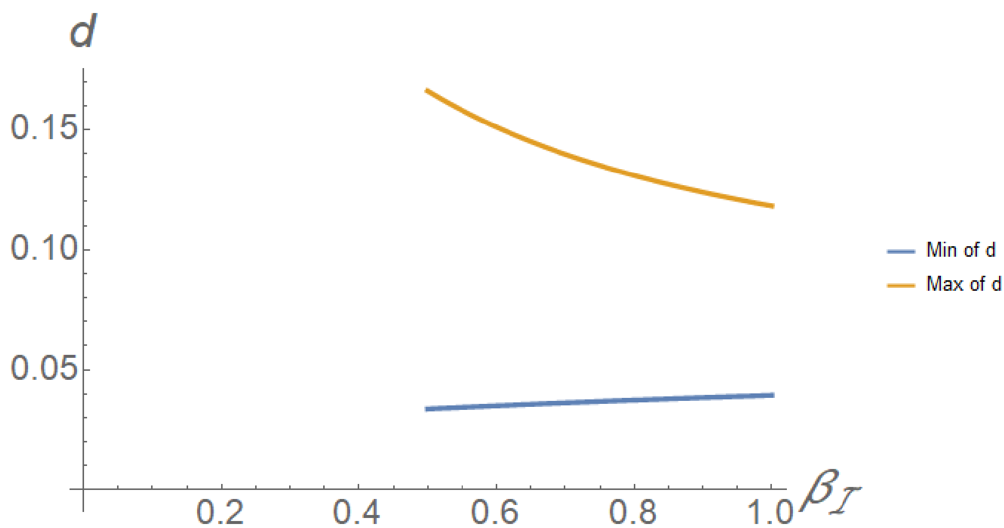

Figure 19: Discrimination Level ($\beta_{\mathcal{A}} = 0.5, m = 0.05$)

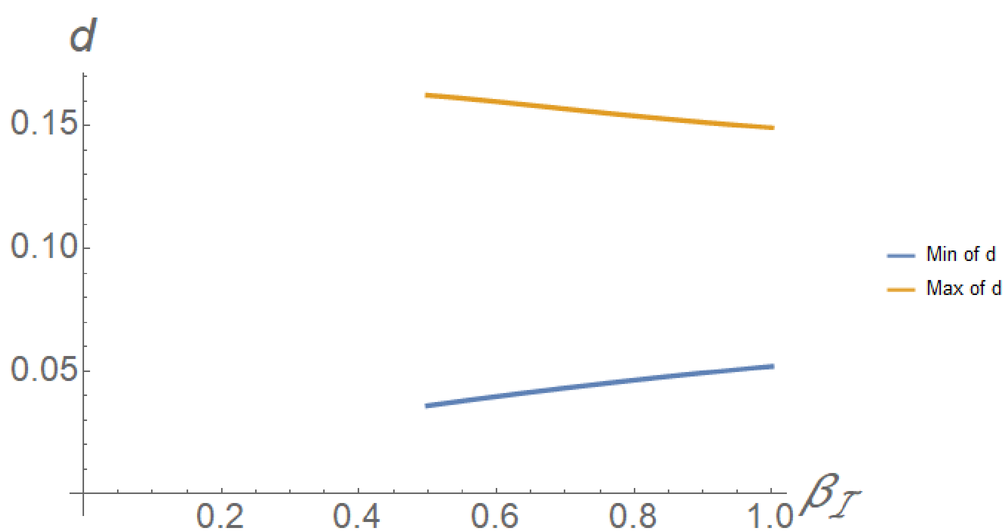

Figure 20: Discrimination Level ($\beta_{\mathcal{A}} = 0.5, m = 0.3$)

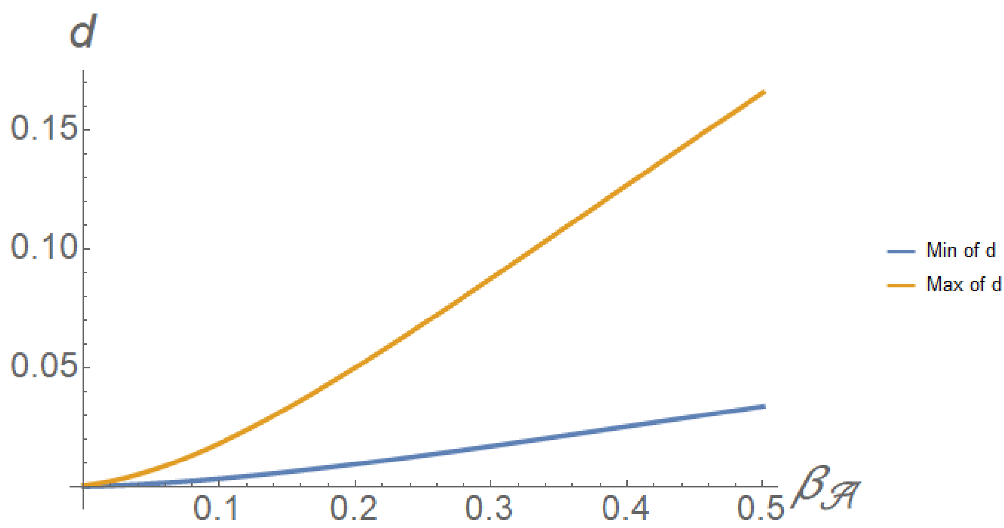

Figure 21: Discrimination Level ($\beta_{\mathcal{I}} = 0.5, m = 0.05$)

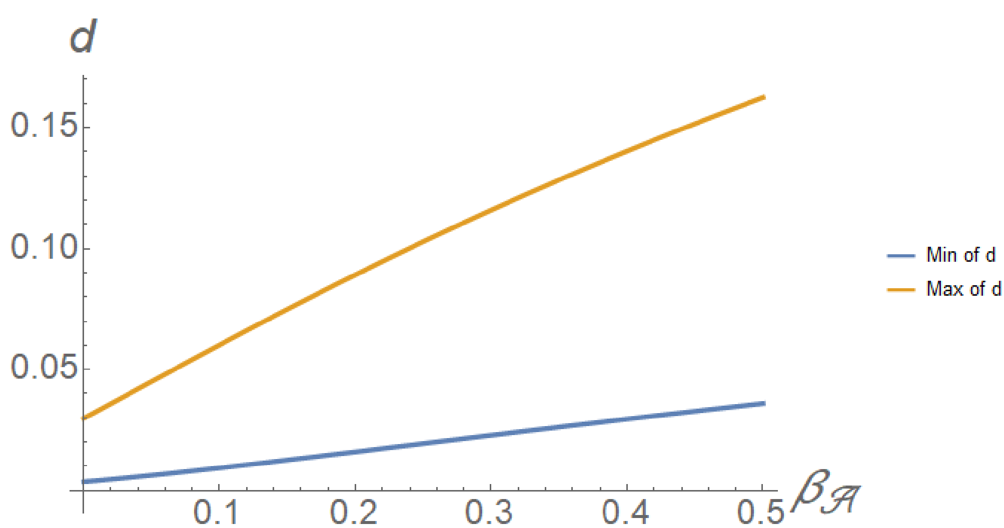

Figure 22: Discrimination Level ($\beta_{\mathcal{I}} = 0.5, m = 0.3$)

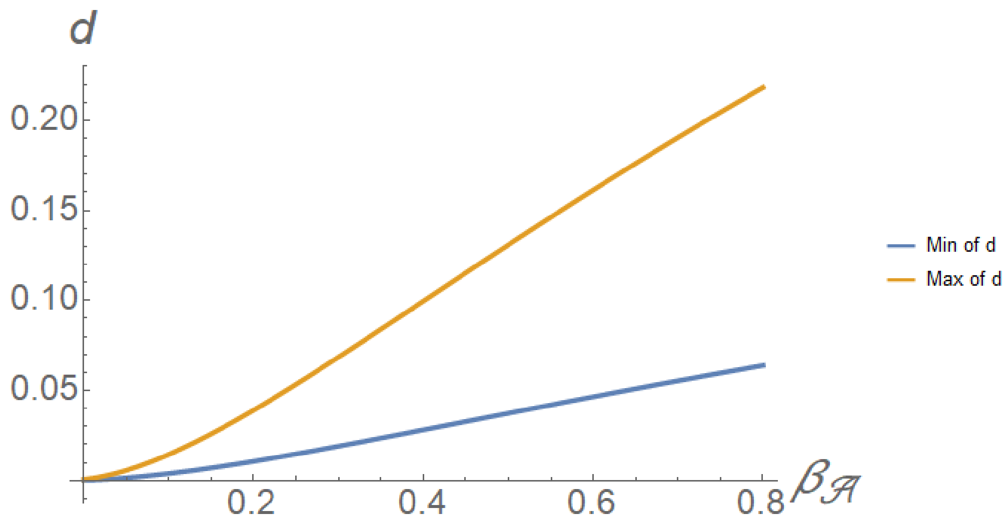

Figure 23: Discrimination Level ($\beta_{\mathcal{I}} = 0.8, c = 0.05$)

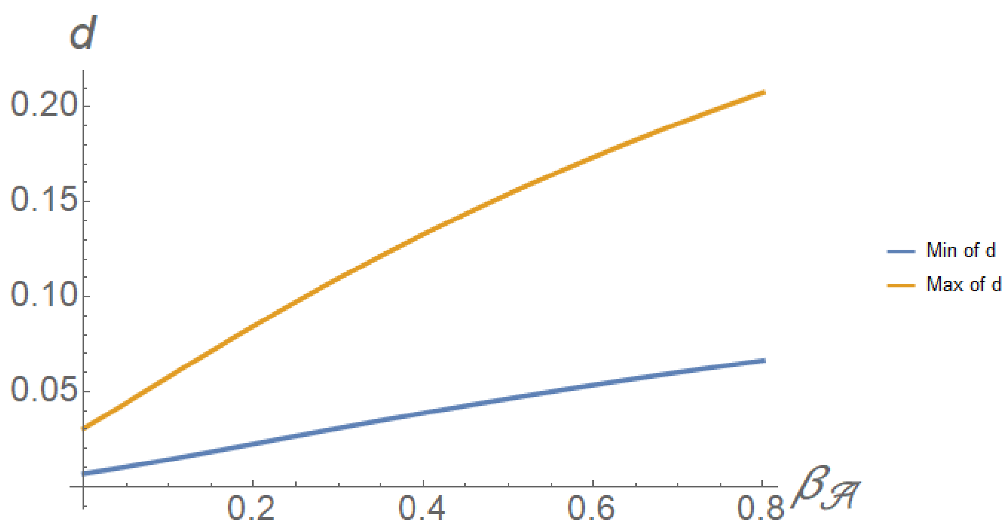

Figure 24: Ability Cutoff ($\beta_{\mathcal{I}} = 0.8, m = 0.3$)

Appendix Proofs

Proof of Lemma 1. For any $d \in \mathbb{R}_+$, we want to show that there exist a θ^c such that the equilibrium exist.

If $f(m)g \leq f(1-m)$, then no one will assimilate, so $a_i = 0$ for all $i \in N_I$.

If $f(m)g < f(1-m)$ and $d = 0$, then everyone will assimilate, so $a_i = 1$ for all $i \in N_I$.

If $f(m)g < f(1-m)$ and $d > 0$:

Suppose the equilibrium exist, then it must be that θ^c satisfies the following condition:

$$f(1-m+m(1-\theta^c))s_A s_{\theta^c} - d(1-\theta^c) = f(m\theta^c)s_I s_{\theta^c}$$

Here $s_A = \frac{(1-m)/2+mg(1-\theta^c)(1+\theta^c)/2}{1-m+m(1-\theta^c)}$ and $s_I = \frac{1}{2}g\theta^c$

Simplify the equation we can get:

$$\begin{aligned}
&(f(1-m\theta^c) - f(m\theta^c))mg^2(\theta^c)^3 + (f(m\theta^c)g^2 + 2dm)(\theta^c)^2 \\
&\quad - (f(1-m\theta^c)(1-m+mg)g + 2d + 2dm)\theta^c + 2d = 0
\end{aligned}$$

Denote the left-hand side of the equation as $F(\theta^c)$, then $F(0) = 2d > 0$ and $F(1) = (1-m)(f(m)g - f(1-m))g < 0$. So there exist $\theta^c \in (0,1)$ such that $F(\theta^c) = 0$ by Intermediate Value Theorem.

Calculate the derivative of F

$$\begin{aligned}
F'(\theta) = &[f(1-m\theta) - f(m\theta)]mg^2 3\theta^2 + [f'(1-m\theta)(-m) - f'(m\theta)m]mg^2\theta^3 \\
&+ [2dm + f(m\theta)g^2]2\theta + f'(m\theta)mg^2\theta^2 \\
&- f(1-m\theta)(1-m+mg)g - 2d(1+m) - f'(1-m\theta)(-m)(1-m+mg)g\theta \\
= &f(1-m\theta)[3mg^2\theta^2 - (1-m+mg)g] \\
&+ f(m\theta)[-3mg^2\theta^2 + 2g^2\theta] \\
&f'(1-m\theta)(-m)[mg^2\theta^3 - (1-m+mg)g\theta] \\
&f'(m\theta)m[-mg^2\theta^3 + g^2\theta^2] \\
&2d(2m\theta - 1 - m) \\
= &f(1-m\theta)[3mg^2\theta^2 - (1-m+mg)g] + f(m\theta)[2g^2\theta - 3mg^2\theta^2] \\
&f'(1-m\theta)m[(1-m+mg)g\theta - mg^2\theta^3] + f'(m\theta)m[g^2\theta^2 - mg^2\theta^3] \\
&+ 2d(2m\theta - 1 - m)
\end{aligned}$$

■

Proof of Lemma 2. For the majority group $\mathcal{A}$, they would like to maximize $f(1-m\theta)s_A$.

$$f(1-m\theta)\frac{(1-m)/2+mg(1-\theta^2)/2}{1-m\theta}=f(1-m\theta)\frac{1-m+mg(1-\theta^2)}{2(1-m\theta)}$$

Denote it $H(\theta)$,then

$$\begin{aligned}
H'(\theta) &= \frac{1}{2}[f'(1-m\theta)(-m)\frac{1-m+mg(1-\theta^2)}{1-m\theta} \\
&\quad + f(1-m\theta)\frac{mg(-2\theta)(1-m\theta)-(1-m+mg(1-\theta^2))(-m)}{(1-m\theta)^2}] \\
&= \frac{m}{2}[f'(1-m\theta)\frac{mg\theta^2-(1-m+mg)}{1-m\theta} \\
&\quad + f(1-m\theta)\frac{1-m+mg(1-\theta^2)-2g\theta(1-m\theta)}{(1-m\theta)^2}] \\
&= \frac{m}{2(1-m\theta)^2}[-f'(1-m\theta)(1-m\theta)(1-m+mg(1-\theta^2)) \\
&\quad + f(1-m\theta)[1-m+mg(1-\theta^2)-2g\theta(1-m\theta)]] \\
&= \frac{m}{2(1-m\theta)^2}[f(1-m\theta)[1-m+mg-2g\theta+mg\theta^2] \\
&\quad - f'(1-m\theta)(1-m\theta)(1-m+mg(1-\theta^2))]
\end{aligned}$$

$H(\theta)$ is continuous so it must achieve maximum on $[0,1]$.

$$\begin{aligned}
H'(0) &= \frac{m}{2}[-f'(1)(1-m+mg)+f(1)(1-m+mg)] \\
&= \frac{m}{2}(1-m+mg)(f(1)-f'(1))>0
\end{aligned}$$

Since $f(0)=0$, $f''()<0$ and $f'(0)<1$, so $f(1)>f'(1)$

$$\begin{aligned}
H'(1) &= \frac{m}{2(1-m)^2}[-f'(1-m)(1-m)(1-m)+f(1-m)(1-m-2g(1-m))] \\
&= \frac{m}{2(1-m)}[-f'(1-m)(1-m)+f(1-m)(1-2g)]<0
\end{aligned}$$

0 and 1 are not local maximum for $H(\theta)$, so $H(\theta)$ achieve maximum on $(0,1)$.

■

Proof of Lemma 4. Denote

$$P(\theta)=f(1-m\theta)[1-m+mg-2g\theta+mg\theta^2]-f'(1-m\theta)(1-m\theta)(1-m+mg(1-\theta^2))$$

$$
\begin{aligned}
P'(\theta) = {} & f(1-m\theta)[-2g+2mg\theta] + f'(1-m\theta)(-m)[1-m+mg-2g\theta+mg\theta^2] \\
& - f'(1-m\theta)[(-m)(1-m+mg-mg\theta^2)+(1-m\theta)(-2mg\theta)] \\
& - f''(1-m\theta)(-m)(1-m\theta)(1-m+mg-mg\theta^2) \\
= {} & f(1-m\theta)[2mg\theta-2g] + f'(1-m\theta)4mg\theta(1-m\theta) \\
& + f''(1-m\theta)m(1-m\theta)(1-m+mg-mg\theta^2) \\
= {} & -(1-m\theta)\left[f(1-m\theta)2g - f'(1-m\theta)4mg\theta - f''(1-m\theta)m(1-m+mg-mg\theta^2)\right]
\end{aligned}
$$

$$
\begin{aligned}
f(1-m\theta) - f'(1-m\theta)2m\theta & \geq f(1-m) - f'(1-m)2m \\
& \geq f'(1-m)(1-m) - f'(1-m)2m \\
& = f'(1-m)(1-3m)
\end{aligned}
$$

So $P'(\theta) < 0$ for $\theta \in [0,1]$ when $m \leq \frac{1}{3}$.

Also, $H'(\theta) = \frac{m}{2(1-m\theta)^2}P(\theta)$ so that $H'(\theta)$ is strictly decreasing when $\theta \in [0,1]$. Thus, $H(\theta)$ has a unique maximum point.

■

Proof of Lemma 5.

For $f(x) = \frac{1}{2} - \frac{1}{2}(1-x)^2 = x - \frac{1}{2}x^2$, We have

$$
\begin{aligned}
2F(\theta) = {} & [1-2m\theta]mg^2\theta^3 + [4dm + (2m\theta - m^2\theta^2)g^2]\theta^2 \\
& - [(1-m^2\theta^2)(1-m+mg)g + 4d(1+m)]\theta + 4d \\
= {} & -3m^2g^2\theta^4 + [3mg^2 + (1-m+mg)m^2g]\theta^3 \\
& + 4dm\theta^2 - (1-m+mg)g\theta - 4d(1+m)\theta + 4d
\end{aligned}
$$

$$
\begin{aligned}
2F'(\theta) = {} & -12m^2g^2\theta^3 + \theta^2[9mg^2 + 3(1-m+mg)m^2g] \\
& - (1-m+mg)g + 4d(2m\theta - 1 - m)
\end{aligned}
$$

We want to show that $F'(\theta) < 0$ for $\theta \in [0,1]$. Since $m < \frac{1}{2}$, we know that $2m\theta - 1 - m < 0$ for $\theta \in [0,1]$. Denote

$$
G(\theta) = -12m^2g^2\theta^3 + \theta^2[9mg^2 + 3(1-m+mg)m^2g] - (1-m+mg)g
$$

So $2F'(\theta) = G(\theta) + 4d(2m\theta - 1 - m)$

$$G'(\theta) = -36m^2g^2\theta^2 + 6[3mg^2 + (1 - m + mg)m^2g]\theta$$

$G'(\theta) = 0$ when $\theta = 0$ or $\frac{3g+(1-m+mg)m}{6mg}$. $m < \frac{1}{2}$ so that $\frac{3g+(1-m+mg)m}{6mg} > 1$, then $G'\theta) \geq 0$ when $\theta \in [0, 1]$.

$$G(0) = -(1 - m + mg)g < 0$$

$$\begin{aligned} G(1) &= -12m^2g^2 + 9mg^2 + 3(1 - m + mg)m^2g - (1 - m + mg)g \\ &= [(-3m^3 + 3m^2 + m - 1) + (3m^3 - 12m^2 + 8m)g]g \\ &= [(-3m^2 + 1)(m - 1) + m(3(m - 2)^2 - 4)]g < 0 \end{aligned}$$

$G(\theta)$ is strictly increasing for $\theta \in [0, 1]$ and $G(0) < 0$, $G(1) < 0$, so $G(\theta) < 0$ for $\theta \in [0, 1]$.

Thus $2F'(\theta) = G(\theta) + 4d(2m\theta - 1 - m)$, then $F'(\theta) < 0$ for $\theta \in [0, 1]$. So the solution of $F(\theta) = 0$ is unique.

■

Proof of Proposition 2.

For $f(x) = x - \frac{1}{2}x^2$, we have

$$\begin{aligned} H'(\theta) &= \frac{m}{2(1 - m\theta)^2}\left[\frac{1}{2}(1 - m^2\theta^2)[1 - m + mg - 2g\theta + mg\theta^2] - m\theta(1 - m\theta)(1 - m + mg(1 - \theta^2))\right] \\ &= \frac{m}{4(1 - m\theta)}\left[(1 + m\theta)(1 - m + mg - 2g\theta + mg\theta^2) - 2m\theta(1 - m + mg - mg\theta^2)\right] \\ &= \frac{m}{4(1 - m\theta)}[(1 - m + mg)(1 + m\theta) + (g\theta)(m\theta) - 2(g\theta) + (g\theta)(m\theta)^2 - 2(g\theta)(m\theta) \\ &\qquad - 2m\theta(1 - m + mg) + 2(g\theta)(m\theta)^2] \\ &= \frac{m}{4(1 - m\theta)}[(1 - m + mg)(1 - m\theta) + g\theta(3m\theta + 2)(m\theta - 1)] \\ &= \frac{m}{4}[1 - m + mg - g\theta(3m\theta + 2)] \\ &= -\frac{m}{4}[3mg\theta^2 + 2g\theta - (1 - m + mg)] \end{aligned}$$

The we know that $\theta^c = \frac{-g+\sqrt{g^2+3mg(1-m+mg)}}{3mg}$. Then $d(1 - \theta^c) = \theta^c g(f(m(A))s_A -$

$f(m(I))s_I)$.

$$\begin{aligned}
d &= \frac{g\theta^c}{2(1-\theta^c)(1-m\theta^c)}[1-m+mg(1-(\theta^c)^2)-g\theta^c(1-m\theta^c)] \\
&= \frac{g^2(\sqrt{g^2+3mg(1-m+mg)}-g)[3m(1-m+mg)+g-\sqrt{g^2+3mg(1-m+mg)}]}{2m(3mg+g-\sqrt{g^2+3mg(1-m+mg)})(4g-\sqrt{g^2+3mg(1-m+mg)})}
\end{aligned}$$

Which is unique.

■